different voltage levels to each state. This connection was the step necessary to apply Boole’s theory to the practical design of computers. As a result, Boole is considered one of the founding fathers of computers and information technology.

William Arthur Atkins (with Philip Edward Koth)
Sharon Cade

See also Babbage, Charles • Computers, Evolution of Electronic • Mathematical Devices, Early • Mathematical Devices, Mechanical

Bibliography


Bouncing Ball, Measurement of a

When a ball bounces, different mathematical models can describe what happens. If the ball bounces in place several times, a geometric sequence* or exponential* model describes the maximum height that the ball attains in relation to the number of bounces. For any single bounce, a quadratic* model describes the height of the ball at any point in time.

Exponential Model: Maximum Height

When examining the maximum height a bouncing ball attains, one ignores external factors such as air resistance. A ball bounced in place recovers a certain percentage of its original height. For example, suppose a ball that recovers 70 percent of its height is dropped from 200 feet. The maximum height it reaches after its first bounce is 70 percent of 200 feet, or 140 feet. After the second bounce, it reaches a height of 70 percent of 140 feet, or 98 feet. In similar fashion, the ball continues to rebound to a height that is 70 percent of...
the highest point of the previous bounce. The following graph illustrates these maximum heights.

Because each successive maximum height is found by multiplying the previous height by the same nonzero value, this is a geometric sequence. The maximum height can also be expressed as an exponential function, with the domain* restricted to whole numbers*. For this example, the maximum heights attained are shown on the next page—second column, with values rounded to the nearest tenth of a foot.

**Infinite Bouncing**  Because the height of each successive bounce continues to be 70 percent of a positive number, the ball’s bounce, in theory, will never reach a zero height. In practice, however, the ball loses energy and does eventually stop bouncing.

It is possible to calculate the total distance the ball travels in this theoretical world of endless bounces. Initially, the ball travels 200 feet. It then bounces up 140 feet and falls 140 feet, a total of 280 feet. It then bounces up 98 feet and falls 98 feet, a total of 196 feet. This pattern continues.

After the initial 200 feet, an infinite geometric series* appears: $280 + 196 + 137.2 + 96.04 + \ldots$. Summing this series with the formula

$$S = \frac{a_1}{1 - r}$$

results in $\frac{280}{1 - 0.7} - 933\frac{1}{3}$ feet.

Adding the initial 200 feet, the total distance that the ball travels is $1,133\frac{1}{3}$ feet.

**Quadratic Model: Height of Single Bounce**

To consider the height of the ball at any given point in time, one again assumes a recovery of 70 percent, an initial height of 200 feet, and no air resistance. To determine how long it takes for the ball to fall from each height, one uses a formula from physics that relates an object’s height to the length of time it has been falling, $h = -16t^2 - v_0t + h_0$.

Here 16 is half the rate of acceleration due to gravity in feet per second squared, $v_0$ is the initial velocity (0 in this case since the ball is dropped), and $h_0$ is the initial height (200 feet). Note that it takes the same amount of time to reach a height as it takes to fall from it (the following
table of values is rounded to tenths and hundredths, which means the results may not be exact).

### REALITY CHECK

Both the geometric sequence (exponential model) and the quadratic (parabolic) model discussed in this entry are theoretical. An interesting activity is to conduct an experiment, collect the data, and compare the experimental data with these theoretical models.

This relationship is a series of parabolas* of decreasing height (because the maximum height decreases) and decreasing width (because the time between bounces decreases). Algebraically, if \( h \) represents the height and \( t \) represents the time, the first parabola can be expressed as

\[
h = 200 - 16t^2, \quad 0 \leq t < 3.54.
\]

Between the first and second bounces, the height can be expressed as

\[
h = 140 - 16(t - 6.5)^2, \quad 3.54 \leq t < 9.45.
\]

The 6.5 (3.54 time down on first bounce plus 2.96, time up on the second bounce) is when the highest point is achieved (halfway between the two bounces), and the 9.45 seconds is derived from adding 3.54 seconds to the time between the first and second bounces (5.92)—there is a rounding error since the table only goes to the hundredths place. It is at this time that the ball bounces again. One could continue deriving these equations in similar fashion. The following figure shows this series of parabolic arcs.

<table>
<thead>
<tr>
<th>BOUNCE</th>
<th>MAXIMUM HT (feet)</th>
<th>TM UP (seconds)</th>
<th>TM DOWN (seconds)</th>
<th>TM BETWEEN BOUNCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
<td>0</td>
<td>3.54</td>
<td>3.54</td>
</tr>
<tr>
<td>1</td>
<td>200 \times 0.7^2 - 140</td>
<td>2.96</td>
<td>2.96</td>
<td>5.92</td>
</tr>
<tr>
<td>2</td>
<td>200 \times 0.7^2 - 98</td>
<td>2.47</td>
<td>2.47</td>
<td>4.95</td>
</tr>
<tr>
<td>3</td>
<td>200 \times 0.7^2 - 88.6</td>
<td>2.07</td>
<td>2.07</td>
<td>4.14</td>
</tr>
<tr>
<td>4</td>
<td>200 \times 0.7^2 - 80.0</td>
<td>1.73</td>
<td>1.73</td>
<td>3.46</td>
</tr>
<tr>
<td>5</td>
<td>200 \times 0.7^2 = 33.6</td>
<td>1.45</td>
<td>1.45</td>
<td>2.90</td>
</tr>
<tr>
<td>x</td>
<td>200 \times 0.7^2 \sqrt{12.5 \times 0.7^2}</td>
<td>\sqrt{12.5 \times 0.7^2}</td>
<td>\sqrt{12.5 \times 0.7^2}</td>
<td>2\sqrt{12.5 \times 0.7^2}</td>
</tr>
</tbody>
</table>

* parabola a conic section; the set of all points such that the distance from any given point from a fixed point (called the focus) is equal to the distance to a fixed straight line (called the directrix)

---

**A Forward-Moving Bouncing Ball** Now assume that a ball, held at 200 feet, is thrown horizontally at the rate of 1 foot per second. The ball accelerates downward, acted upon by gravity, as it continues to travel horizontally, maintaining its horizontal rate of 1 foot per second. Because it accelerates downward at the same rate as a ball bounced in place, the heights of the two balls are identical at any point in time. Consequently, the graphs of their heights in relation to time are identical.
If the horizontal speed is other than 1, the scaling* must be adjusted.

Bob Horton
Sharon Cade

See also Exponential Growth and Decay • Quadratic Formula and Equations

Bibliography


Brain, Human

The human brain has been compared to many things in an attempt to understand how it works. For instance, similarities have been pointed out between the connections in the brain and a large telephone switchboard. More recently, frequent parallels have been drawn between the brain and modern digital computers. This comparison may be particularly appropriate because computers are being used in many applications that until recently were largely thought to be beyond the capability of machines and the exclusive province of the human brain.

The Computer Analogy

Computers are now being used in tasks from playing chess at a grand master level to visually recognizing what is being “seen” through video cameras in specific applications. The human brain’s comparison to a computer is the best analogy thus far, although there are important differences.

Digital computers are built from logic circuits* or “logic gates” that produce a predictable output based on the inputs. The inputs are generally one of two selected voltages, which are typically thought of as the binary values 0 (zero) and 1 (one). Gates may produce a logical 1 output if all of the inputs are 1 (an AND function), if any of the inputs are 1 (an OR function), or if the inputs can be inverted, producing a 1 only if the input is 0 (a NOT function).

From these simple functions, more complex operations can be built by interconnecting logic gates, including circuits capable of adding numbers and memory devices capable of storing information. A digital computer may be built from millions of individual gates for its central